Topic: Tangent lines

Question: Find the equation of the tangent line at the given point.

$$y = 3x^2 - 6x + 1$$

at $(1, -2)$

Answer choices:

A $y = -2$

B $x + y = -2$

C $y = 2$

D $x - y = 2$

© Krista King Math

Solution: A

Find the slope of the tangent line by differentiating the function and evaluating its value at the indicated point.

$$y' = 6x - 6$$

At $(1, -2)$,

$$y' = 6(1) - 6$$

$$y' = 0$$

so the slope of the tangent line at the given point is 0.

Using the point-slope formula for the equation of a line,

$$y - y_1 = m(x - x_1)$$

we get

$$y - (-2) = 0(x - 1)$$

$$y + 2 = 0$$

$$y = -2$$

© Krista King Math

Topic: Tangent lines

Question: Find the equation of the tangent line at the given point.

$$y = \frac{1}{x^2 + 1}$$

at $\left(1, \frac{1}{2}\right)$

Answer choices:

A $x + 2y = 2$

B $x - y = 1$

C $2x + y = 2$

D $x - 2y = 2$

© Krista King Math

Solution: A

Differentiating the function at the indicated point will give us the slope of the tangent line.

$$y' = \frac{-2x}{(x^2+1)^2}$$

At $\left(1, \frac{1}{2}\right)$,

$$y' = \frac{-2(1)}{\left[(1)^2+1\right]^2}$$

$$y' = -\frac{1}{2}$$

so the slope of the tangent line is $-1/2$.

Using the point-slope formula for the equation of a line,

$$y - y_1 = m(x - x_1)$$

we get

$$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$$

$$y - \frac{1}{2} = -\frac{1}{2}x + \frac{1}{2}$$

$$2y - 1 = -x + 1$$

$$x + 2y = 2$$

© Krista King Math

Topic: Tangent lines

Question: Find the horizontal tangent line(s) of the function.

$$f(x) = x^3 - x - 3$$

on the interval $-1 \leq x \leq 1$

Answer choices:

A 0

B $\pm\frac{\sqrt{3}}{3}$

C $\pm\sqrt{3}$

D ± 3

© Krista King Math

Solution: B

The horizontal tangent lines of a function exist where $f'(x) = 0$. To find the horizontal tangent lines of a function you take the derivative of the function, set the derivative equal to zero and then solve for all possible values of x. Remember, if an interval is given, only the horizontal tangent lines that fall inside the interval are considered and any other answers can be discarded.

Let's start with our original function

$$f(x) = x^3 - x - 3$$

Now let's use power rule to take the derivative.

$$f'(x) = 3x^2 - 1$$

Remember, the horizontal tangent lines exist at $f'(x) = 0$, so we'll set our derivative equal to 0.

$$0 = 3x^2 - 1$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}}$$

$$x = \pm\frac{1}{\sqrt{3}}$$

© Krista King Math

$$x = \pm \frac{\sqrt{3}}{3}$$

The function $f(x) = x^3 - x - 3$ on the interval $-1 \leq x \leq 1$ has two horizontal tangent lines at $x = \pm \frac{\sqrt{3}}{3}$.

© Krista King Math